Comparing with mortality models, nuptiality and fertility models are more complex and less developed. The complexity mostly comes from (1) fertility may differ by marital status, (2) marriage, divorce, and widowhood may occur more than once (death must occur only once). Thus the model fit the data less closely than mortality models.

Coale-McNeil nuptiality model

$$g(a) = (0.174/k) \cdot \exp\left\{-4.411 \cdot \exp\left\{-(0.309/k)(a - a_0)\right\}\right\}$$

a: age, $g(a)$: the risk of first marriage at age a, a_0: earliest age at which there are a significant number of first marriages, k: parameter for the shape of the curve. Ranges of a_0 are from 12 (past India) to 18 (developed countries), because it's minimum age of marriage. Ranges of k are from 0.2 (very rapid increase rate of marriage after a_0) to 1.7 (very slow).

k=0.5 indicates a rate of marriage that is double that in the standard population (k=1.0).

Modified version of Coale-McNeil model (1972) include the scale parameter C, the proportion of women who eventually marry within her whole life. The model is,

$$g(a) = 0.19465 \cdot C \cdot \exp\left\{-0.174 \cdot ((a - a_0)/k - 6.06) - \exp\left\{-(0.288 \cdot ((a - a_0)/k - 6.06)\right\}\right\}$$

In fmsb package, the function CM(scale=0.8, a0=18, k=2)$G gives proportion ever married for ages 10 to 60. fitCM() can automatically fit the model to actual data of proportion ever married.
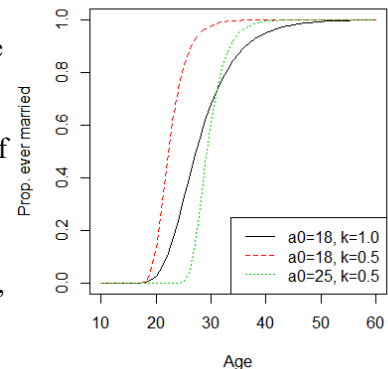
Coale-Trussell fertility model

$$r(a) = M \cdot n(a) \cdot \exp\left\{m \cdot \nu(a)\right\}$$

a: age, $r(a)$: proportion of marital fertility occurring at age a, M: scale parameter, $n(a)$: natural fertility pattern (given as table), m: the extent of fertility control, $\nu(a)$: the pattern of fertility decline with aging (also given as table). In fmsb package, CT(M=1, m=0) function gives age-specific marital fertilities for ages from 12 to 49.

Combining Coale-McNeil nuptiality model with Coale-Trussell marital fertility model makes overall fertility model shown below (where $G(a)$ indicates the proportion ever married).

$$f(a) = G(a) \cdot n(a) \cdot \exp\left\{m \cdot \nu(a)\right\}$$

Gompertz relational fertility model

It resembles to Brass's logit model as relational model life table because of using transformation for linear regression by standard schedule. This model uses Gompit transformation for the cumurative proportions of age-specific fertility rates among TFR.

Let p ASFR/TFR, Gompit(p) = -ln(-ln p). p=exp(-exp(-Gompit)).
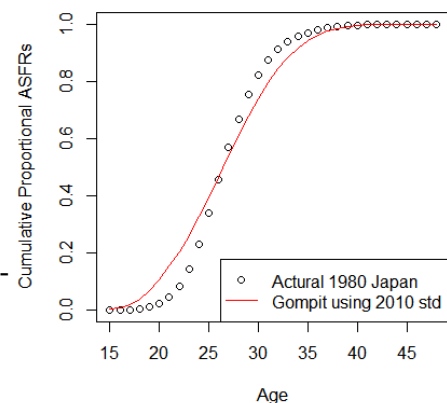
Other models of fertility

Brass's fertility polynomial: f(x) = c(x-s)(s+33-x)^2

x: age, f(x): cumulative fertility, c: level parameter, s: age at which fertility begins, s+13.2 is the mean age of childbearing.

Hadwiger's model:

$$\mathrm{ASFR}(x) = \left(\frac{ab}{c}\right)\left(\frac{c}{x}\right)^{\frac{3}{2}} \exp\left\{-b^2\left(\frac{c}{x} + \frac{x}{c} - 2\right)\right\}$$

where a, b, and c are arbitrary, but a correlates with TFR, b with peak height of ASFRs, c with mean age of childbearing.

See the R code, will be given as **http://minato.sip21c.org/demography-special/Chap14.R**

(Appendix) "Proximate determinants of fertility" models were considered by Bongaarts J, Wood JW, et al. Original Idea: Remote determinants (cultural, social, economic, …) → Proximate determinants (Intermediate variables: proportion of married women, contraceptive use, breastfeeding, induced abortion, intercourse frequency, spontaneous fetal loss, sterility due to diseases) → Fertility (TFR, parity, completed family size, …)
See, http://ocw.jhsph.edu/courses/FamilyPlanning/pdfs/session2.1notes.pdf
TFR = TF (total fecundity) x Cm (non-marriage) x Cc (contraception) x Ca (induced abortion) x Ci (lactational infecundability)