Rational decision-making: Game theory and causal inference

- Game theory (Von Neumann J and Morgenstern O, 1944) assumed rational human behavior (as same as classic economics) under perfect information
  - Rational judgement to maximize one's own benefit considering counterpart's rational strategy in 2 players, without "cooperation" → Not always optimum as a whole
    - Zero-sum game → Mini-max theorem
    - Extension to non zero-sum game such as Prisoner's dillemma → Nash equilibrium
  - Pareto optimum: Resource sharing can be improved Pareto efficiently, with cooperation
  - Under imperfect information, rational judgement is more difficult
  - Open Yale Courses: Game theory https://oyc.yale.edu/economics/econ-159
- Causal inference (Please read Rothman et al. (2023) Epidemiology: An Introduction 3rd Ed. Chapter 3, freely available from companion site*)
  * https://global.oup.com/us/companion.websites/9780197807842/
  - Logically infer the interrelationship between causes and effect
  - Checklist is not enough. For most checklist, exceptions exist.
  - Pearl's causal diagram: DAG (Directed Acyclic Graph)
    - https://cran.r-project.org/web/packages/ggdag/vignettes/intro-to-dags.html
    - Pearl, J. (2009). An Introduction to Causal Inference. UCLA: Department of Statistics & Data Science, UCLA. https://escholarship.org/uc/item/9f06b02k
  - Robins' g-method

Zero-sum game and minimax strategy

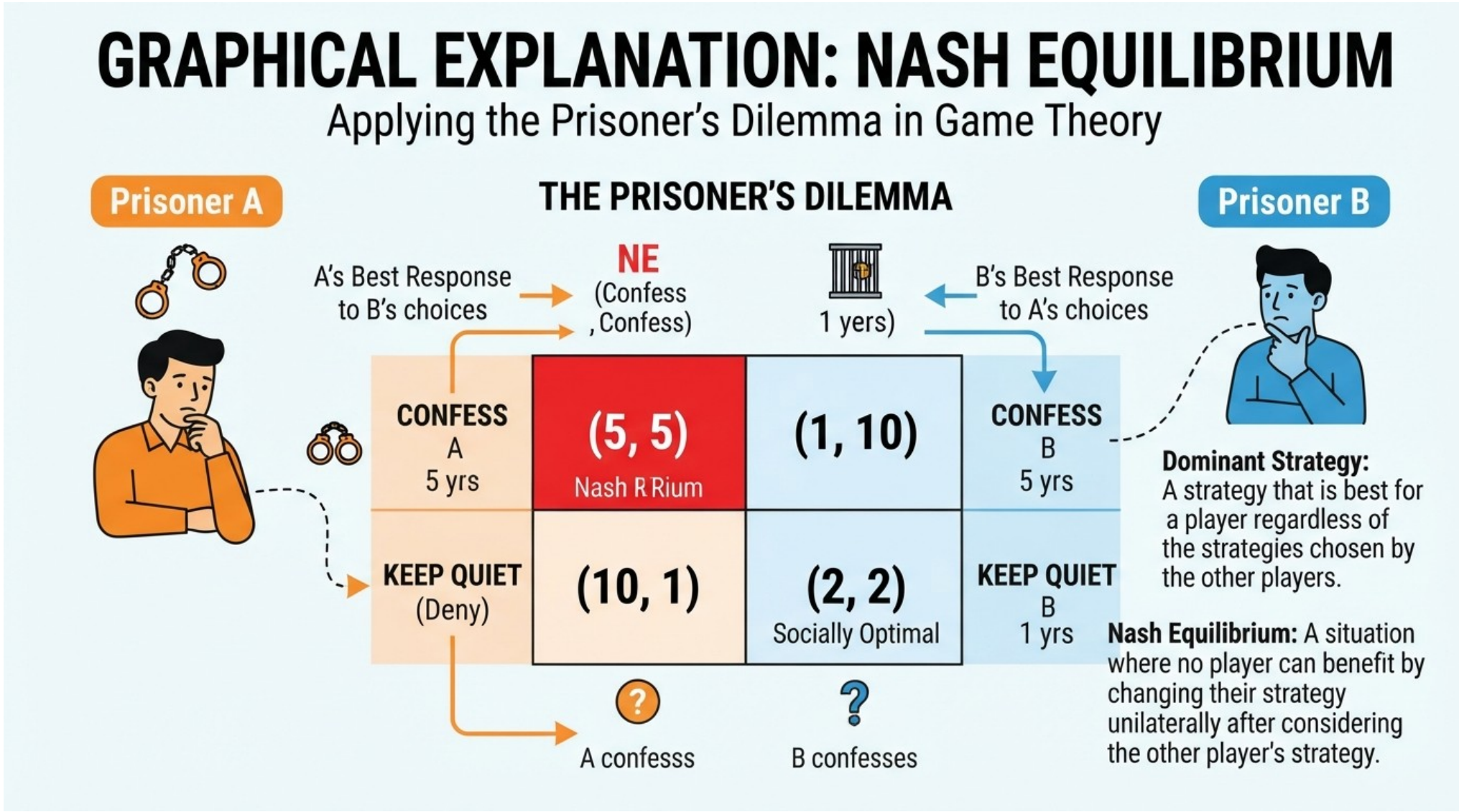
https://doi.org/10.1016/B0-12-227410-5/00272-6

- Numbers show the earnings by the player 1 (same as loss by the player 2, thus zero-sum)
- Player 1 can choose from D-F
  - E maximizes minimum earning (=2) regardless with player 2's choice
  - Choosing E is minimax strategy for player 1
- Player 2 can choose from A-C
  - A maximizes minimum earning (=2) regardless with player 1's choice
  - Choosing A is minimax strategy for player 2
- E and A is optimal choice, that is also Nash equilibrium

	A	B	C
D	1	3	5
E	2	4	3
F	0	1	-3

Nash equilibrium for non zero-sum game

(* Illustration below is provided by Gemini)



Pareto optimum with cooperation

(* Graphical presentation is provided by Gemini)

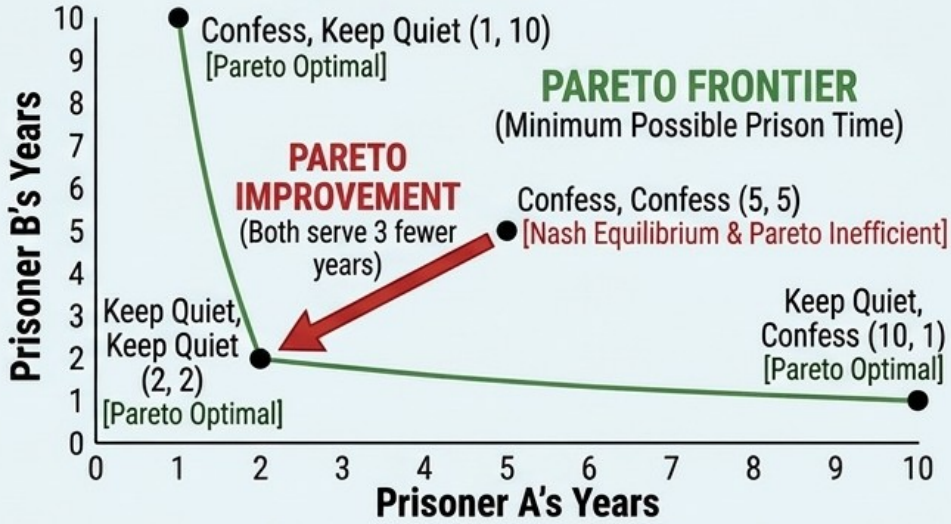
PARETO OPTIMALITY IN THE PRISONER'S DILEMMA (YEARS IN PRISON)

PAYOFF MATRIX: PRISON YEARS (A, B)

	Confess	Keep Quiet
Confess	(5, 5)	(1, 10)
Keep Quiet	(10, 1)	(2, 2)

Payoffs are years in prison. Lower numbers are better (less prison time).

VISUALIZING THE PAYOFF SPACE



Origin (0,0) = Best Outcome (0 years)

KEY DEFINITIONS:

1. **PARETO OPTIMALITY:** An outcome where no player can get fewer prison years without forcing the other player to serve more years.
2. **PARETO IMPROVEMENT:** Moving to an outcome where at least one player serves fewer years and neither serves more.
3. **NASH EQUILIBRIUM:** (5, 5) Confess/Confess — neither suspect can reduce their own sentence by changing strategy alone.

ANALYSIS OF THE DILEMMA:

In this game, mutual silence (2, 2) is Pareto Optimal because neither suspect can get a shorter sentence without increasing the other's sentence (to 10 years). However, individual self-interest drives both to confess, landing at the Nash Equilibrium (5, 5). This outcome is Pareto Inefficient because moving to (2, 2) is a clear Pareto Improvement that cuts both sentences from 5 years to 2 years.

Causal diagrams

- Most imputation techniques need the knowledge to clarify the relationships among variables each other → Causal diagrams are useful
- In Rothman's text, Figure 15.1 (source: Lipsky and Greenland, 2022) https://www.researchgate.net/publication/358932233_Causal_Directed_Acyclic_Graphs
  - E is associated with O if there is an open (unblocked) path between them
  - The association may be mediated via one or more other variables (M)
  - E and O are directionally connected: E is cause of O
  - If we control for M in the analysis, we block the path from E to O through M, direct effect (E → O) remains
  - E and O are independent if there is no path connecting them or every path connecting them is blocked (= directionally separated)

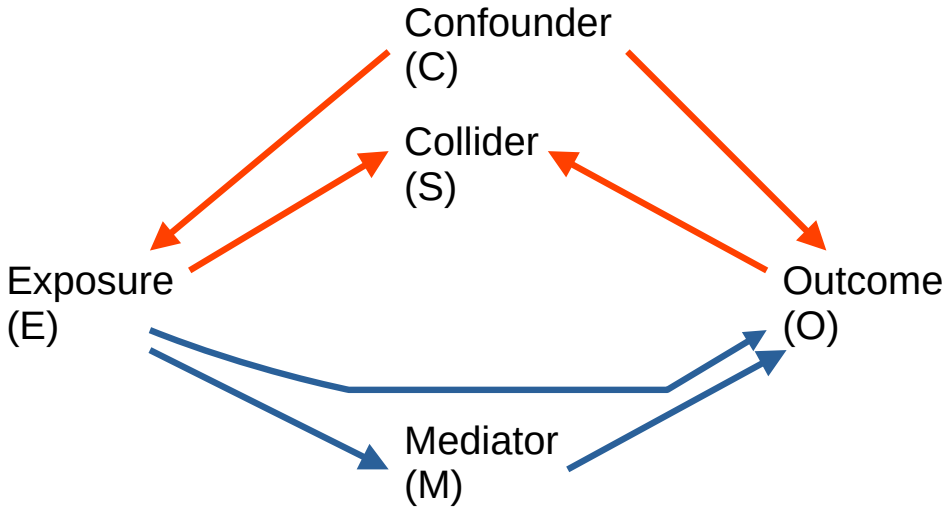
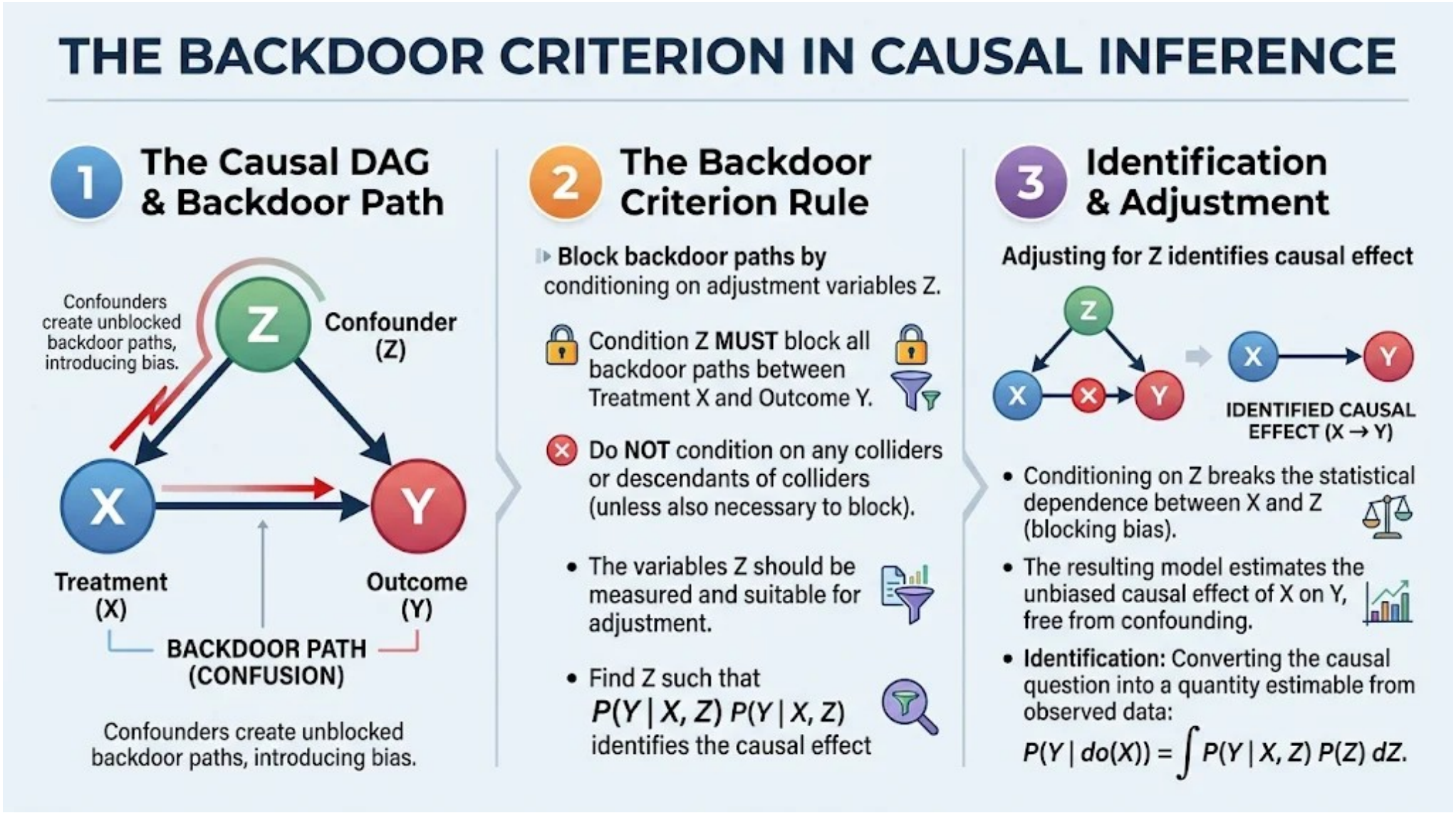


Figure 15.1 Directed acyclic graph (**DAG**) illustrating directed (or causal) and nondirected (or bias) paths.

- Nondirected paths are potential sources of bias.
  - When the third variable is associated with E and is a risk factor of O, it's confounding: E ← C → O (So called "backdoor" path, unblocked)
  - When a variable is a consequence of both E and O, it's a collider: E → S ← O, this path is blocked by the collider. If this variable is ignored, no bias, but if it's adjusted (=opening backdoor) in analysis, it causes bias.

Backdoor criterion

(* Graphical presentation is provided by Gemini)



Graphical Connection, Association, and Causation

- Collider bias explains the observations that smoking appeared to be protective against serious COVID-19: Suffering from smoking-related illness and severe COVID-19 both increase risk of hospitalization. **Those 2 risk factors are truly unassociated**.
- Assuming there are only 2 reasons for hospitalization and anyone with either risk gets hospitalized
- Table 15.1: A total of 5+45+95 people are hospitalized, no association
- Table 15.2: Wihout 855 with neither risk factor. Strong negative association exists → All negative for smoking have COVID-19, and vise versa.
- By conditioning on the **collider** of hospitalization, we may open a backdoor path between smoking and COVID-19, also called as Berksonian bias.

Table 15.1 Lack of association in general population

	Smoking	No Smoking	Total
COVID-19 infection	5	45	50
No COVID-19 infection	95	855	950
Total	100	900	1000

Table 15.2 Illustrating association among hospitalized patients

	Smoking	No Smoking	Total
COVID-19 infection	5	45	50
No COVID-19 infection	95	0	95
Total	100	45	145

Time-dependent variables

- Exposures change over time → **time-varying exposures**
  - Components of diet or chronic medication use occur daily or vary seasonally
  - Radiation exposure during a mammogram occur infrequently and sporadically
- A simplistic way is to ignore the time variation: Compare ever exposed vs never exposed → Such a dichotomous exposure definition cannot capture detailed history of exposure
- By taking the exposure history into account in the exposure definition (eg. cumulative measures of smoking such a pack-years), it's improved
- Potential confounders: unpredictably changing blood pressure, BMI, cholesterol, physical activity, exposures to sunlight, … The exposure at some time affecting the subsequent exposure (**Figure 15.2, from Rothman's text**)
- C ↔ E: C is confounder (and thus should be controlled) and causal intermediate (and thus should not be controlled) → Dilemma

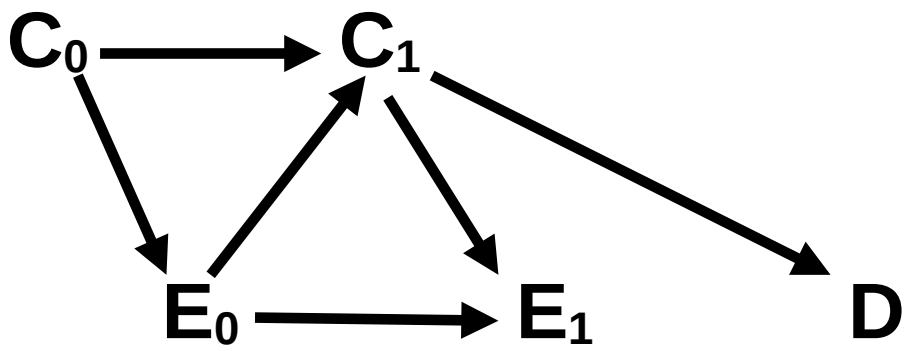


Figure 15.2 Directed acyclic graph (**DAG**) illustrating time-varying confounding.

- Consider a study evaluating the effect of asthma rescue mediation (E) on pulmonary function (D).
- The effect of medication is confounded by recent severity of asthma symptoms (C).
- Negative feedback bias the effect of treatment to result in apparent no benefit of the medication
- To solve it, g-methods (g-formula, marginal structural models, structural nested models) can be applied: These allow for stepwise feedback between time-varying treatments and time-varying confounders.

g-methods

- In Japanese, please see
  https://www.m.u-tokyo.ac.jp/sph/wp-content/uploads/2024/09/3ee194aec0a51beb45cd166123592ac1.pdf
- Naimi AI et al. (2017) An introduction to g methods. *Int J Epidemiol* 46(2): 756-762. https://doi.org/10.1093/ije/dyw323
- g-methods estimate contrasts of potential outcomes under a less restrictive set of assumptions than standard regression methods.
  - **Inverse probability weighting** generates a pseudo-population in which exposures are independent of confounders, enabling estimation of marginal structural model parameters.
  - **g-estimation** exploits the conditional independence between the exposure and potential outcomes to estimate structural nested model parameters. In R, gesttools package can be used. https://doi.org/10.1353/obs.2022.0003
  - The **g-formula** models the joint density of the observed data to generate potential outcomes under different exposure scenarios. In R, gfoRmula (https://doi.org/10.1016/j.patter.2020.100008) package is available.